Esercizi sulla sintassi del Calcolo dei Predicati

Esercizio 1

S'individuino, in ciascuna delle seguenti formule, le occorrenze libere e quelle vincolate di tutte le variabili.

1.1 $\exists y\,(\forall x \exists z \exists k\,(A\,(x,y) \to B\,(x,k,z,t)) \vee \exists x\,(C\,(x,y,z) \vee \exists s D\,(s,z)))$

1.2 $\exists x\,(\forall y \exists z \exists k\,(A\,(x,t,y) \to B\,(x,k,z,s)) \vee \exists y\,(C\,(x,y,z) \vee \exists s G\,(s,z)))$

1.3 $\exists x \forall y\,(\exists z \exists k\,(A\,(x,t,y) \to B\,(x,k,z,s)) \vee \exists z\,(C\,(x,y,z) \vee \exists s G\,(s,z)))$

1.4 $(\exists x A\,(x,y,z) \vee (\forall z\,(B\,(y) \vee B\,(z)) \wedge \exists x C\,(x,t,s))) \leftrightarrow \forall s D\,(s,x,k)$

1.5 $(\exists x A\,(x,y,z) \vee \forall z\,(B\,(y,z) \vee C\,(z))) \vee \exists x \forall k\,(D\,(x,t,s) \leftrightarrow \forall s E\,(s,x,k))$

1.6 $(\exists z A\,(x,y,z) \to \forall y\,(B\,(y,z) \vee C\,(z))) \wedge \forall x\,(\exists s D\,(x,k,s) \vee \forall k E\,(s,x,k))$

1.7 $(\exists z A\,(x,y,z) \wedge \forall y\,(B\,(y,z) \to C\,(z))) \vee \exists x\,(\exists s D\,(x,k,s) \wedge \forall k E\,(s,x,k))$

1.8 $(\exists z A\,(x,y,z) \wedge \forall y\,(B\,(y,z) \to C\,(z))) \vee \exists x\,(\exists s D\,(x,k,s) \wedge \forall k E\,(s,x,k))$

1.9 $(\exists x A\,(x,y,z) \vee \forall z\,(B\,(y,z) \vee C\,(z))) \vee \exists x \forall k\,(C\,(x,t,s) \leftrightarrow \forall s D\,(s,x,k))$

1.10 $(\exists x A\,(x,y,z) \vee \forall z\,(B\,(y,z) \vee C\,(z))) \vee \exists x \forall k\,(C\,(x,t,s) \leftrightarrow \forall s D\,(s,x,k))$

Esercizio 2

Per ciascuna delle seguenti formule si scrivano in Forma Normale Prenessa sia la formula stessa che la sua negata.

2.1 $\varphi \equiv \forall x \forall y((A(x) \vee B(x,y,z)) \to \forall t C(t,s,x))$

2.2 $\varphi \equiv \exists x(\forall z(A(x,z) \to B(x,y,z)) \to \exists t C(t,s,x))$

2.3 $\varphi \equiv \exists x \forall y A(x,y) \leftrightarrow \exists z \neg B(z)$

2.4 $\varphi \equiv (\exists x \forall y A(x,y) \vee \exists y \neg B(y)) \wedge (\forall x C(x) \to \exists z D(z))$

2.5 $\varphi \equiv (\forall x \exists y \neg A(x,y) \to \exists y \neg B(y)) \wedge \forall x(C(x) \wedge \neg \exists z D(z))$

2.6 $\varphi \equiv \forall x((\exists y A(x,y) \wedge \forall y \neg B(y) \wedge \exists z C(z)) \to \exists z D(z))$

Soluzioni

Esercizio 1

Sottolineiamo una volta le occorrenze libere e due volte quelle vincolate.

1.1 $\exists y \left(\forall x \exists z \exists k \left(A\left(\underline{\underline{x}}, \underline{\underline{y}}\right) \rightarrow B\left(\underline{\underline{x}}, \underline{\underline{k}}, \underline{\underline{z}}, \underline{t}\right)\right) \vee \exists x \left(C\left(\underline{\underline{x}}, \underline{\underline{y}}, \underline{z}\right) \vee \exists s D\left(\underline{\underline{s}}, \underline{z}\right)\right)\right)$

1.2 $\exists x \left(\forall y \exists z \exists k \left(A\left(\underline{\underline{x}}, \underline{t}, \underline{\underline{y}}\right) \rightarrow B\left(\underline{\underline{x}}, \underline{\underline{k}}, \underline{\underline{z}}, \underline{s}\right)\right) \vee \exists y \left(C\left(\underline{\underline{x}}, \underline{\underline{y}}, \underline{z}\right) \vee \exists s G\left(\underline{\underline{s}}, \underline{z}\right)\right)\right)$

1.3 $\exists x \forall y \left(\exists z \exists k \left(A\left(\underline{\underline{x}}, \underline{t}, \underline{\underline{y}}\right) \rightarrow B\left(\underline{\underline{x}}, \underline{\underline{k}}, \underline{\underline{z}}, \underline{s}\right)\right) \vee \exists z \left(C\left(\underline{\underline{x}}, \underline{\underline{y}}, \underline{\underline{z}}\right) \vee \exists s G\left(\underline{\underline{s}}, \underline{\underline{z}}\right)\right)\right)$

1.4 $\left(\exists x A\left(\underline{\underline{x}}, \underline{y}, \underline{z}\right) \vee \left(\forall z \left(B\left(\underline{y}\right) \vee B\left(\underline{\underline{z}}\right)\right) \wedge \exists x C\left(\underline{\underline{x}}, \underline{t}, \underline{s}\right)\right)\right) \leftrightarrow \forall s D\left(\underline{\underline{s}}, \underline{x}, \underline{k}\right)$

1.5 $\left(\exists x A\left(\underline{\underline{x}}, \underline{y}, \underline{z}\right) \vee \forall z \left(B\left(\underline{y}, \underline{\underline{z}}\right) \vee C\left(\underline{\underline{z}}\right)\right)\right) \vee \exists x \forall k \left(D\left(\underline{\underline{x}}, \underline{t}, \underline{s}\right) \leftrightarrow \forall s E\left(\underline{\underline{s}}, \underline{\underline{x}}, \underline{\underline{k}}\right)\right)$

1.6 $\left(\exists z A\left(\underline{x}, \underline{y}, \underline{\underline{z}}\right) \rightarrow \forall y \left(B\left(\underline{\underline{y}}, \underline{z}\right) \vee C\left(\underline{z}\right)\right)\right) \wedge \forall x \left(\exists s D\left(\underline{\underline{x}}, \underline{k}, \underline{\underline{s}}\right) \vee \forall k E\left(\underline{s}, \underline{\underline{x}}, \underline{\underline{k}}\right)\right)$

1.7 $\left(\exists z A\left(\underline{x}, \underline{y}, \underline{\underline{z}}\right) \wedge \forall y \left(B\left(\underline{\underline{y}}, \underline{z}\right) \rightarrow C\left(\underline{z}\right)\right)\right) \vee \exists x \left(\exists s D\left(\underline{\underline{x}}, \underline{k}, \underline{\underline{s}}\right) \wedge \forall k E\left(\underline{s}, \underline{\underline{x}}, \underline{\underline{k}}\right)\right)$

1.8 $\left(\exists z A\left(\underline{x}, \underline{y}, \underline{\underline{z}}\right) \wedge \forall y \left(B\left(\underline{\underline{y}}, \underline{z}\right) \rightarrow C\left(\underline{z}\right)\right)\right) \vee \exists x \left(\exists s C\left(\underline{\underline{x}}, \underline{k}, \underline{\underline{s}}\right) \wedge \forall k D\left(\underline{s}, \underline{\underline{x}}, \underline{\underline{k}}\right)\right)$

1.9 $\left(\exists x A\left(\underline{\underline{x}}, \underline{y}, \underline{z}\right) \vee \forall z \left(B\left(\underline{y}, \underline{\underline{z}}\right) \vee C\left(\underline{\underline{z}}\right)\right)\right) \vee \exists x \forall k \left(C\left(\underline{\underline{x}}, \underline{t}, \underline{s}\right) \leftrightarrow \forall s D\left(\underline{\underline{s}}, \underline{\underline{x}}, \underline{\underline{k}}\right)\right)$

1.10 $\left(\exists x A\left(\underline{\underline{x}}, \underline{y}, \underline{z}\right) \vee \forall z \left(B\left(\underline{y}, \underline{\underline{z}}\right) \vee C\left(\underline{\underline{z}}\right)\right)\right) \vee \exists x \forall k \left(C\left(\underline{\underline{x}}, \underline{t}, \underline{s}\right) \leftrightarrow \forall s D\left(\underline{\underline{s}}, \underline{\underline{x}}, \underline{\underline{k}}\right)\right)$

Esercizio 2

2.1 φ_{FNP} :

$$\begin{aligned}
&\forall x\forall y((A(x)\lor B(x,y,z))\to\forall tC(t,s,x))\\
&\equiv\forall x\forall y(\neg(A(x)\lor B(x,y,z))\lor\forall tC(t,s,x))\\
&\equiv\forall x\forall y\forall t((\neg A(x)\land\neg B(x,y,z))\lor C(t,s,x))
\end{aligned}$$

$(\neg\varphi)_{FNP}$:

$$\begin{aligned}
&\neg[\forall x\forall y\forall t((\neg A(x)\land\neg B(x,y,z))\lor C(t,s,x))]\\
&\equiv\exists x\exists y\exists t\neg((\neg A(x)\land\neg B(x,y,z))\lor C(t,s,x))\\
&\equiv\exists x\exists y\exists t(\neg(\neg A(x)\land\neg B(x,y,z))\land\neg C(t,s,x))\\
&\equiv\exists x\exists y\exists t((A(x)\lor B(x,y,z))\land\neg C(t,s,x))
\end{aligned}$$

2.2 φ_{FNP} :

$$\begin{aligned}
&\exists x(\forall z(A(x,z)\to B(x,y,z))\to\exists tC(t,s,x))\\
&\equiv\exists x(\neg\forall z(\neg A(x,z)\lor B(x,y,z))\lor\exists tC(t,s,x))\\
&\equiv\exists x\exists z\exists t(\neg(\neg A(x,z)\lor B(x,y,z))\lor C(t,s,x))\\
&\equiv\exists x\exists z\exists t((A(x,z)\land\neg B(x,y,z))\lor C(t,s,x))
\end{aligned}$$

$(\neg\varphi)_{FNP}$:

$$\begin{aligned}
&\neg[\exists x\exists z\exists t((A(x,z)\land\neg B(x,y,z))\lor C(t,s,x))]\\
&\equiv\forall x\forall z\forall t\neg((A(x,z)\land\neg B(x,y,z))\lor C(t,s,x))\\
&\equiv\forall x\forall z\forall t(\neg(A(x,z)\land\neg B(x,y,z))\land\neg C(t,s,x))\\
&\equiv\forall x\forall z\forall t((\neg A(x,z)\lor B(x,y,z))\land\neg C(t,s,x))
\end{aligned}$$

2.3 φ_{FNP} :

$$\begin{aligned}
&\exists x\forall yA(x,y)\leftrightarrow\exists z\neg B(z)\\
&\equiv(\exists x\forall yA(x,y)\to\exists z\neg B(z))\land(\exists z\neg B(z)\to\exists x\forall yA(x,y))\\
&\equiv(\exists x\forall y\neg A(x,y)\lor\exists z\neg B(z))\land(\neg\exists z\neg B(z)\lor\exists x\forall yA(x,y))\\
&\equiv(\exists x\forall y\neg A(x,y)\lor\exists z\neg B(z))\land(\forall zB(z)\lor\exists x\forall yA(x,y))\\
&\equiv(\exists x\forall y\neg A(x,y)\lor\exists z\neg B(z))\land(\forall cB(c)\lor\exists a\forall bA(a,b))\\
&\equiv\exists x\forall y\exists z\exists a\forall b\forall c((\neg A(x,y)\lor\neg B(z))\land(B(c)\lor A(a,b)))
\end{aligned}$$

$(\neg\varphi)_{FNP}$:

$$\begin{aligned}
&\neg[\exists x\forall y\exists z\exists a\forall b\forall c((\neg A(x,y)\lor\neg B(z))\land(B(c)\lor A(a,b)))]\\
&\equiv\forall x\exists y\forall z\forall a\exists b\exists c\neg((\neg A(x,y)\lor\neg B(z))\land(B(c)\lor A(a,b)))\\
&\equiv\forall x\exists y\forall z\forall a\exists b\exists c(\neg(\neg A(x,y)\lor\neg B(z))\lor\neg(B(c)\lor A(a,b)))\\
&\equiv\forall x\exists y\forall z\forall a\exists b\exists c((A(x,y)\land B(z))\lor(\neg B(c)\land\neg A(a,b)))
\end{aligned}$$

2.4 φ_{FNP} :

$$\begin{aligned}
&(\exists x\forall yA(x,y)\lor\exists y\neg B(y))\land(\forall xC(x)\to\exists zD(z))\\
&(\exists x\forall yA(x,y)\lor\exists y\neg B(y))\land(\neg\forall xC(x)\lor\exists zD(z))\\
&\equiv(\exists x\forall yA(x,y)\lor\exists a\neg B(a))\land(\neg\forall bC(b)\lor\exists zD(z))\\
&\equiv(\exists x\forall yA(x,y)\lor\exists a\neg B(a))\land(\exists b\neg C(b)\lor\exists zD(z))\\
&\equiv\exists x\forall y\exists a\exists b\exists z((A(x,y)\lor\neg B(a))\land(\neg C(b)\lor D(z)))
\end{aligned}$$

$(\neg\varphi)_{FNP}$:

$$\begin{aligned}
&\neg[\exists x\forall y\exists a\exists b\exists z((A(x,y)\vee\neg B(a))\wedge(\neg C(b)\vee D(z)))]\\
&\equiv \forall x\exists y\forall a\forall b\forall z\neg((A(x,y)\vee\neg B(a))\wedge(\neg C(b)\vee D(z)))\\
&\equiv \forall x\exists y\forall a\forall b\forall z(\neg(A(x,y)\vee\neg B(a))\vee\neg(\neg C(b)\vee D(z)))\\
&\equiv \forall x\exists y\forall a\forall b\forall z((\neg A(x,y)\wedge B(a))\vee(C(b)\wedge\neg D(z)))
\end{aligned}$$

2.5 φ_{FNP} :

$$\begin{aligned}
&(\forall x\exists y\neg A(x,y)\to\exists y\neg B(y))\wedge\forall x(C(x)\wedge\neg\exists zD(z))\\
&\equiv(\neg(\forall x\exists y\neg A(x,y))\vee\exists y\neg B(y))\wedge\forall x(C(x)\wedge\neg\exists zD(z))\\
&\equiv(\exists x\forall yA(x,y)\vee\exists y\neg B(y))\wedge\forall x(C(x)\wedge\forall z\neg D(z))\\
&\equiv(\exists x\forall yA(x,y)\vee\exists a\neg B(a))\wedge\forall b(C(b)\wedge\forall z\neg D(z))\\
&\equiv\exists x\forall y\exists a\forall b\forall z((A(x,y)\vee\neg B(a))\wedge(C(b)\wedge\neg D(z)))\\
&\equiv\exists x\forall y\exists a\forall b\forall z((A(x,y)\vee\neg B(a))\wedge C(b)\wedge\neg D(z))
\end{aligned}$$

$(\neg\varphi)_{FNP}$:

$$\begin{aligned}
&\neg[\exists x\forall y\exists a\forall b\forall z((A(x,y)\vee\neg B(a))\wedge C(b)\wedge\neg D(z))]\\
&\equiv\forall x\exists y\forall a\exists b\exists z\neg((A(x,y)\vee\neg B(a))\wedge C(b)\wedge\neg D(z))\\
&\equiv\forall x\exists y\forall a\exists b\exists z(\neg(A(x,y)\vee\neg B(a))\vee\neg C(b)\vee D(z))\\
&\equiv\forall x\exists y\forall a\exists b\exists z((\neg A(x,y)\wedge B(a))\vee\neg C(b)\vee D(z))
\end{aligned}$$

2.6 φ_{FNP} :

$$\begin{aligned}
&\forall x((\exists yA(x,y)\wedge\forall y\neg B(y)\wedge\exists zC(z))\to\exists zD(z))\\
&\equiv\forall x(\neg(\exists yA(x,y)\wedge\forall y\neg B(y)\wedge\exists zC(z))\vee\exists zD(z))\\
&\equiv\forall x((\neg\exists yA(x,y)\vee\neg\forall y\neg B(y)\vee\neg\exists zC(z))\vee\exists zD(z))\\
&\equiv\forall x(\forall y\neg A(x,y)\vee\exists yB(y)\vee\forall z\neg C(z)\vee\exists zD(z))\\
&\equiv\forall x(\forall y\neg A(x,y)\vee\exists aB(a)\vee\forall z\neg C(z)\vee\exists bD(b))\\
&\equiv\forall x\forall y\exists a\forall z\exists b(\neg A(x,y)\vee B(a)\vee\neg C(z)\vee D(b))
\end{aligned}$$

$(\neg\varphi)_{FNP}$:

$$\begin{aligned}
&\neg[\forall x\forall y\exists a\forall z\exists b(\neg A(x,y)\vee B(a)\vee\neg C(z)\vee D(b))]\\
&\equiv\exists x\exists y\forall a\exists z\forall b\neg(\neg A(x,y)\vee B(a)\vee\neg C(z)\vee D(b))\\
&\equiv\exists x\exists y\forall a\exists z\forall b(A(x,y)\wedge\neg B(a)\wedge C(z)\wedge\neg D(b))
\end{aligned}$$